ORIGINAL PAPER

Estimating the effects of activists in two-party and multi-party systems: comparing the United States and Israel

**Norman Schofield · Christopher Claassen ·
Ugur Ozdemir · Alexei Zakharov**

Abstract   This article presents an electoral model where activist groups contribute resources to their favored parties. These resources are then used by the party candidates to enhance the electoral perception of their quality or *valence*. We construct an empirical model of the United States presidential election of 2008 and employ the electoral perception of the character traits of the two candidates. We use a simulation technique to determine the local Nash equilibrium, under vote share maximization, of this model. The result shows that the unique vote-maximizing equilibrium is one where the two candidates adopt convergent positions, close to the electoral center. This result conflicts with the estimated positions of the candidates in opposed quadrants of the policy space. The difference between estimated positions and equilibrium positions allows us to estimate the influence of activist groups on the candidates. We compare this estimation with that of Israel for the election of 1996, and show that vote maximization leads low valence parties to position themselves far from the electoral origin. We argue that these low valence parties in Israel will be dependent on support of radical activist groups, resulting in a degree of political fragmentation.

N. Schofield (✉) · C. Claassen · U. Ozdemir
Center in Political Economy, Washington University, 1 Brookings Drive,
Saint Louis, MO 63130, USA
e-mail: schofield.norman@gmail.com

C. Claassen
e-mail: claassen@wustl.edu

U. Ozdemir
e-mail: uozdemir@artsci.wustl.edu

A. Zakharov
State University-Higher School of Economics Moscow, Moscow, Russia
e-mail: 3ax@mail.ru

🖄 Springer

1 Introduction

This article offers a unified model of the electoral process in order to account for a number of general empirical observations about the effects of political institutions. As Duverger (1954) and Riker (1953) have observed, there appears to be a relationship between the electoral rule in place, and the number of political parties in the polity. A highly majoritarian (or plurality) system tends to result in just two parties, while an electoral system based on proportional representation (PR) tends to give a fragmented political structure.[1] Many authors have also argued that there is a relationship between fragmention and the durability of government (Taylor and Herman 1971; Warwick 1994). Other authors have argued that these differing constitutional rules profoundly affect the nature of the policy process (Bawn and Rosenbluth 2005; Persson and Tabellini 2000, 2003).

It is possible that the degree of political fragmentation is a direct consequence of the details of the electoral rule, and the opportunities these provide for strategic voting in the electorate. However, the formal spatial electoral model has not, in our view, been able to offer a plausible account of this relationship. Indeed, as discussed in Schofield (2007a), the extensive literature on formal "deterministic" or "stochastic" vote models tend to suggest that all parties should adopt vote-maximizing positions at the center of the electoral distribution.[2] Such models assume an underlying symmetry in the motivations and dispositions of party leaders, and as a result they are unable to account for the extreme heterogeneity of political configurations observed by Benoit and Laver (2006), for example, in their analysis of party positions in European polities.

In this article, we offer a formal stochastic model of elections that emphasizes the importance of the idea of *valence*. In the standard spatial model, only candidate *positions* matter to voters. However, as Stokes (1963, 1992) has emphasized, the non-policy evaluations, or *valences*, of candidates by the electorate are equally important. Stokes (1963, p. 373) used the term *valence issues* to refer to those that "involve the linking of the parties with some condition that is positively or negatively valued by the electorate." As he observes, "in American presidential elections... it is remarkable how many valence issues have held the center of the stage." We use the stochastic valence electoral model to compare party strategies in Israel, where the electoral system is based on proportional rule, with that of the United States, where the electoral system is highly majoritarian.

We argue that, in the United States, the differences between the valences of the two major presidential candidates are insufficient to force them to adopt divergent positions. Instead, the logic of vote maximization should force convergence to the electoral origin.[3] Since candidates do not converge, we propose a model where activist groups provide the resources that are critical for political success. However, these activists require the candidates to adopt divergent positions in return for political support. In

---

[1] See Laakso and Taagepera (1979) for a formal definition of fragmentation.

[2] See Downs (1957), Riker and Ordeshook (1973), and McKelvey and Patty (2006).

[3] The *electoral origin* is simply that point which is at the mean on all dimensions of the distribution of voter preferred points.

essence, a small number of influential activist groups induce the polarization that has been noted in the U.S.polity.[4]

In contrast, in Israel, as in other polities based on proportional representation, there are significant valence differences between the parties. By themselves, these valence differences are sufficient to force the parties to diverge. Activists may well influence the parties, but this influence appears much less significant than in the United States. More importantly, since small parties may aspire to membership of coalition government, their activist coalitions have no incentive to coalesce. Thus, the relatively fragmented party structure is maintained. In the United States, the greater intensity of competition for activist support means that small activist groups, if they are to have any impact, must join one or other of the major party activist groups. This forces coalescence of the activist groups. We argue that the relationship between political fragmentation and the nature of the electoral system, noted by Riker (1953) and Duverger (1954), is the result of this logic of activist support.

There is a long tradition of argument that interest groups induce policy choices that are non-optimal for the society (Olson 1965; Keefer 2004; Acemoglu and Robinson 2006), but this article is the first, we believe, to develop a formal and empirical vote model that indicates how to estimate the impact of activist groups on the policy stances of political leaders in polities with different electoral systems.[5]

Electoral models involving this notion of valence has formed the basis for recent extensive analyses of British, Canadian and US electoral response by Clarke et al. (2009a) and Clarke et al. (2005, 2009b).[6]

For Britain, they argue that electoral responses

> were a reflection largely of [the] changing perceptions of the decision-making competence of the main political parties and their leaders. At any point in time, [the] preferences were strongly influenced by their perceptions of the capacity of the rival parties—the putative alternative governments of the day—to solve the major policy problems facing the country.

These works have shown that valence, as measured by the perceptions of the character traits of the candidates, or of party leaders, is a key element of election.

Here, we extend the usual spatial model by incorporating these electoral perceptions of candidate character traits in a stochastic model of the 2008 US election. Our purpose is different from the empirical work by Clarke et al. (2009b). Instead of focusing on the electoral response to candidates, we use this extended model to determine the response of candidates to the electoral situation: that is we compute the equilibrium candidate positions in the context of the chosen model.

---

[4] It is of interest that Bernhardt et al. (2009) use a one-dimensional vote model to argue that polarization of party positions, if not too extreme, is welfare enhancing because of the choice that it provides for the electorate.

[5] An early article by Enelow and Hinich (1982) presented a formal model involving valence, though they used the term "non-spatial characteristic" rather than valence, but their model was not related to the impact of interest groups.

[6] See Schofield et al. (2010c) for similar studies of Britain and Canada.

In the stochastic model, a voter's perception of each candidate's traits has a very significant impact on the probability that the voter chooses one candidate or the other. Since these voters are characterized by different preferred policy positions, a candidate's optimal policy position should be a function of the distribution of these correlated voter positions/perceptions.[7] Our simulation of the combined model, based on both position and valence, allows us to estimate what we call *Local Nash equilibria* (LNE) to the vote-maximizing game, as calibrated by the empirical model with the greatest statistical significance.[8]

We found, by simulation of the stochastic model of the US presidential election in 2008 involving perception of candidate traits, that there was a unique local Nash equilibrium very close to the electoral origin.

In contrast, simulation of the multiparty stochastic model of Israel in 1996 found that the local Nash equilibrium were characterized by divergence away from the electoral center.

In Sect. 2 of this article, we briefly sketch our argument about the fundamental differences in these two polities. Section 3 introduces the notion of the *convergence coefficient* which can be used to determine whether candidates or parties should converge to the electoral center when they attempt to maximize vote share. Sections 4 and 5 present the empirical analyses of these two polities. The concluding section emphasizes the differences between the majoritarian electoral system of the United States and the proportional system in Israel, as well as other fragmented polities such as Poland and Turkey, that are highlighted by the formal and empirical analyses. This conclusion mentions other work that has estimated the convergence coefficients for various polities, and suggests these coefficients are related to the degree of political fragmentation in these political systems. Formal definitions for the model are given in Appendix 1. See also the Appendix to Schofield et al. (2011) in this issue, for the definitions of the equilibrium concepts used here.

2 Comparison of the United States and Israel

To provide a brief sketch of the results on the 2008 U.S. election, consider Fig. 1, which presents the distribution of voter preferred points, as obtained from factor analysis of survey responses from the American National Election Study (ANES 2008). This survey allows us to estimate each respondent's *ideal point,* as a way of representing that citizen's responses. The estimated distribution of such points is the *electoral distribution.* We shall refer to the space in which the electoral distribution is embedded as the *factor space*. In formal spatial models, this space is usually known as the policy space. We use the term *factor space* to remind the reader that the basis for the construction of this space is the factor analysis of the survey. In particular, the electoral distribution is directly estimated from the factor analysis.

---

[7] Just as in Clarke et al. (2009b), we use factor analysis of the survey responses to obtain a two-dimensional representation of the voter preferred positions.

[8] Erikson and Romero (1990) used this procedure in an empirical model of the 1988 US Presidential election. However, they estimated local equilibrium positions on a number of separate policy dimensions, rather than in a single multidimensional policy space.
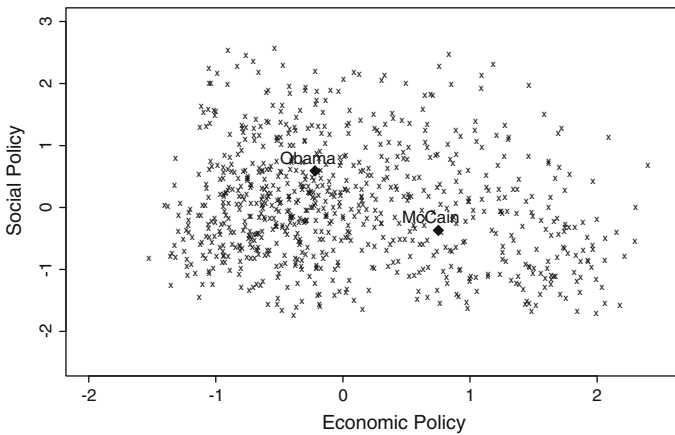
Fig. 1 Distribution of voter ideal points and candidate position in 2008

For any polity, we refer to that point in the factor space which is at the mean in all dimensions of the electoral distribution as *the electoral origin*. The electoral distribution is characterized by variances of the distribution on each axis, as well as the covariance between the various axes.[9]

Figure 1 represents the voter locations in a two-dimensional factor space, so the electoral distribution is given by a symmetric 2 by 2 *covariance matrix*. The x-axis involves economic or redistributive issues, and on this axis the variance of the electoral distribution is 0.80. The y-axis involves social issues, and the variance on this axis is 0.83.[10]

In an empirical model presented below, we estimate Obama's position in the factor space to be $z_{Obama} = (-0.22, +0.75)$, a distance of 0.77 units from the electoral origin.[11] McCain's position in the same factor space was estimated to be $z_{McCain} = (0.59, -0.37)$, a similar distance of 0.69 from the origin, but in a different quadrant of the policy space.[12]

Moreover, the average Democrat voter position was $z_{DEM}^{vote} = (-0.17, +0.36)$[13], while the average Republican voter position was $z_{REP}^{vote} = (0.72, -0.56)$.[14] The survey also gave information on activists of the parties (that is, individuals who contributed money to the parties). Figure 2 shows the distribution of activist positions, clearly very

---

[9] The factor analysis in w dimensions thus gives a symmetric w by w electoral covariance matrix.

[10] The covariance between the two axes is -0.127.

[11] Details of the estimation method are given below. These estimates are the average perceptions of the voters about the candidate positions.

[12] The distance in the factor space between the candidates was 1.38. Since the total electoral variance was 1.63, we term $\sqrt{1.63} = 1.27$ the electoral standard deviation (esd). The two candidates are thus located about 1.08 esd apart.

[13] The variances of the distribution of Democrat partisans ideal points on the two axes were $(0.72, 0.75)$, giving standard errors of the means of $(0.029, 0.03)$.

[14] The variances of the Republican partisans' ideal points were $(0.38, 0.47)$, so the standard errors of these two means were $(0.027, 0.03)$.
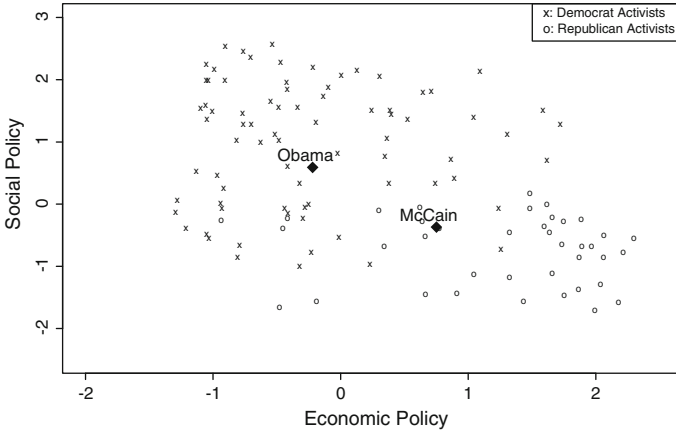
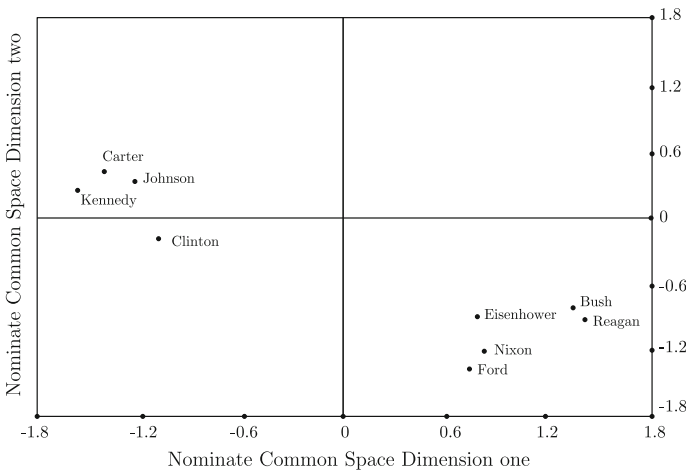**Fig. 2** Distribution of activist ideal points and candidate positions

Fig. 3 Presidents' positions

different from the voter distribution given in Fig. 1. The mean activist positions for the two parties were estimated to be $z_{\text{DEM}}^{\text{act}} = (-0.2, +1.14)$ for the Democrats and $z_{\text{REP}}^{\text{act}} = (1.4, -0.82)$ for the Republicans.

The point to note about these estimates is that the Obama and McCain positions appear significantly different, and some distance from the electoral origin. Obama is located midway on the social axis between Democrat voters and activists, while McCain is more centrist than both Republican voters and activists on the two axes.

The positioning of Democrat presidential candidates in the upper left of the policy space, and Republican candidates in the lower right, has been noted in other empirical work, as suggested by Fig. 3.[15]

Related work (Schofield et al. 2003) has modeled the presidential elections of 1964 and 1980, and argued that such a configuration is a structural characteristic of the US

---

[15] This figure is taken from Schofield (2002). See other related work by Poole and Rosenthal (1984).

polity. Indeed, Miller and Schofield (2003, 2008) and Schofield and Milller (2007) suggest that the US polity is fundamentally two dimensional. They use this feature as the basis for a model of political realignment (Sundquist 1973), as one dimension becomes more important than the other. One purpose of this article is to provide a formal account of what seems to be opposed policy positions offered by the presidential candidates in elections in the United States.

We argue here that the shifts in candidate positions for the two parties over time are insufficient to account for the quite substantial changes in electoral support that occur. Instead our analysis suggests that electoral shifts are primarily the result of changes in the perceptions by the electorate of the candidates. These electoral transformation, in turn, are the consequence of the changing resources available to the candidates. Finally, these are due to the shifting coalition structures among the potential activist groups in the polity.

If this suggestion is correct, then it implies that formal models of elections based on position and valence alone are quite inadequate to account for candidate policy proposals. The following remarks and inferences suggest that any formal model of US elections must expliccitly include activist groups:

 (i) The equilibrium analysis of spatial models of US presidential elections indicates that candidates should converge to positions very close to the electoral origin in order to maximize vote shares.[16]

 (ii) However, estimates of candidate positions indicate that they are located in opposed quadrants of the policy space.

(iii) The incompatibility of the equilibrium locations and the estimated positions can be explained by the influence of activists in US elections.

(iv) Activist influence has increased over time.[17] The recent Supreme Court decision, *Citizens United v. Federal Election Commission*, on Thursday, January 21, 2010, has removed limits on campaign contributions and will likely increase the importance of activist contributions. Dworkin (2010) has called this decision "an unprincipled political act with terrible consequences for the nation." Obama, shortly after, in his State of the Union address declared

> the Supreme Court reversed a century of law [which] I believe will open the floodgates for special interests... to spend without limit in our elections.

 (v) Although the distribution of voter positions may not change dramatically, so the distribution cannot be seen to be polarized, the positions of candidates for office have become more polarized.[18] The system of primaries in US elections is likely to further enhance the influence of activists on candidates.

(vi) Because of this polarization of candidate positions, a shift in the party controlling the presidency will have significant policy implications.

---

[16] This result has also been found by Enelow and Hinich (1989) for earlier U.S. elections.

[17] Indeed, Herrera et al. (2008) observe that spending by parties in federal campaigns went from 58 million dollars in 1976 to over 1 billion in 2004 in nominal terms.

[18] See for example McCarty et al. (2006), and Fiorina et al. (2005).

(vii)   The same argument holds for members of Congress, and we would expect activist influence to increase the degree of polarization in Congress.[19]

(viii) The influence of activists in the strongly majoritarian polity of the United States is the fundamental cause of these policy shifts.

(ix)   Because the winner of the presidential election will generally be located some distance from the electoral center, the policies supported by the President need not be supported by an electoral majority. This phenomenon can be seen with regard to the reform of health care, supported by Obama in 2009/2010. This policy is certainly located in the upper left quadrant of the policy space. As of January 22, 2010, about 39% of the electorate supported the health plan while 55% did not.[20]

(x)   In between elections, diametrically different policy positions will be aggressively supported by opposed lobbying groups. For example, in 2009, health care, pharmaceutical and insurance lobbyists spent approximately $650 million on lobbying itself, and about $210 million on media advertising.[21] The oil and gas industry spent about $560 million.[22]

(xi)   Actual policy choices will depend on complex bargaining between the President and Congress. As the health care issue illustrates, the supramajoritarian voting rule in the Senate will tend to favor the status quo.[23]

(xii)   Activist-induced policy preferences in Congress is extremely heterogenous.[24] This, together with a non-centrist presidential policy position, can result in so-called "gridlock." [25]

(xiii) During 2010, there is an increasing perception in the electorate that Congress has become dysfunctional because of "strident partisanship, unyielding ideology and a corrosive system of campaign financing." [26]. For example, the CNN/Opinion Research Corp. poll, conducted on February 12–15,2010, with 1,023 respondents, found that 86% thought government was "broken." Of these, however, 81% felt it could be "fixed." In fact, gridlock can be overcome, as illustrated by the 62–30 vote in the Senate on February 22 to implement a multi-billion "jobs

---

[19] Conflict between the parties over health care in 2009 and 2010 is just one illustration of this phenomenon.

[20] The surprise victory by Republican Scott Brown over Democrat Martha Coakley in the special election for the Senate seat for Massachusetts, on January 19, 2010, may be indicative of this electoral response, as well as "Tea Party activism."

[21] The pharmaceutical industry was a strong supporter of reform of health care, because of an agreement with Obama to protect the industry's profits.

[22] Tomasky (2010) gives a figure of $3.47 billion for spending by lobbyists in the non election year of 2009, citing data from the Center for Responsive Politics.

[23] Scott Brown's victory in Massachusetts in January, 2010, deprived the Democrats of the 60 seat majority required to overcome the filibuster and push through legislation on health care and other policy issues such as financial reform.

[24] Work by Jeong et al. (2010) estimated the policy positions of US senators with regard to the 2006 immigration reform act and found the Republican senator positions to be very heterogenous, but all clearly in the lower right hand quadrant of the policy space.

[25] This of course contradicts the argument by Bernhardt et al. (2009) that divergence is welfare enhancing.

[26] Indeed, when Evan Bayh, Senator from Indiana, announced in February 2010, he would retire, these were the reasons he gave (Bayh 2010).

creation" program. Gridlock over health care was also broken on March 25, after strenuous efforts by President Barack Obama and House speaker, Nancy Pelosi, when the House voted 220–207 for the health care bill. Republicans had voted unanimously against the legislation, joined by 33 dissident Democrats. The President had signed a draft of the bill, the "Patient Protection and Affordable Care Act" on March 23, and the Senate passed the bill by simple majority of 56 to 43, as required for reconciliation. In July 15, the Senate voted 60–39 for the bill for Reform of Financial Regulation.[27] As of July 2010, there remain four major bills to put through Congress: A Deficit Reduction Act, an Energy Independence and Climate Change Act, an Expanded Trade and Export Act, and a Comprehensive Immigration Act. If these prove impossible to enact because of Republican opposition, the electorate may blame the GOP.

The success of the health care legislation and of reform of financial regulation, together with the signing of the new START arms reduction treaty in Prague on April 8 by Presidents Medvedev and Obama, has certainly increased Obama's international prestige.

However, given the uncertainty surrounding policy choice in the Legislature, it is hardly surprising that voters in the United States doubt that government can be effective. Part of the problem would appear to be the degree of political polarization resulting from the power of interest groups located in the opposed quadrants of the policy space.

We contrast these observations on the difficulties facing the U.S. government with inferences about a polity based on an electoral system using proportional representation, such as Israel. We argue that in such a polity, activist influence is weaker, and policy shifts between different governments will be significantly smaller.

Figure 4 presents a smoothed estimate of the voter distribution, as well as the estimated party positions in Israel in 1996.[28]

The x-axis is designated security, and is defined in terms of attitudes to the PLO. The factor model was normalized with respect to this factor, so the electoral distribution on this axis had a variance of 1.0. The y-axis involves religious attitudes and on this axis the variance was 0.732. Note that in Fig. 4, the estimated positions of the two major parties, Labor and Likud, in the factor space are $(-0.8, -0.3)$ and $(0.4, 0.2)$.[29] In the discussion of Israel elections in Sect. 3 of this article, we argue that the policy positions of coalition governments will depend on whether there is a core party, or one that is centrally located, and large enough to dominate coalition bargaining.

In the election of 2006, a centrist party, Kadima, initially under the leadership of Ariel Sharon, was able to position itself at the electoral origin, and form a coalition

---

[27] This complex bill was 2,300 pages long. Russ Feingold, a Democrat, voted against the bill, because it was not strong enough. Three moderate New England Republicans, Snowe and Collins of Maine, and Scott Brown of Massachusetts, voted for the bill.

[28] The party positions were obtained from expert estimates, and the voter distribution obtained from a survey by Arian and Shamir (1999). See Schofield and Sened (2006).

[29] The distance in the factor space between these two parties was 1.3. Since the total electoral variance was 1.732, the electoral standard deviation (esd) was $\sqrt{1.732} = 1.32$. The two parties were thus located 0.98 esd apart.
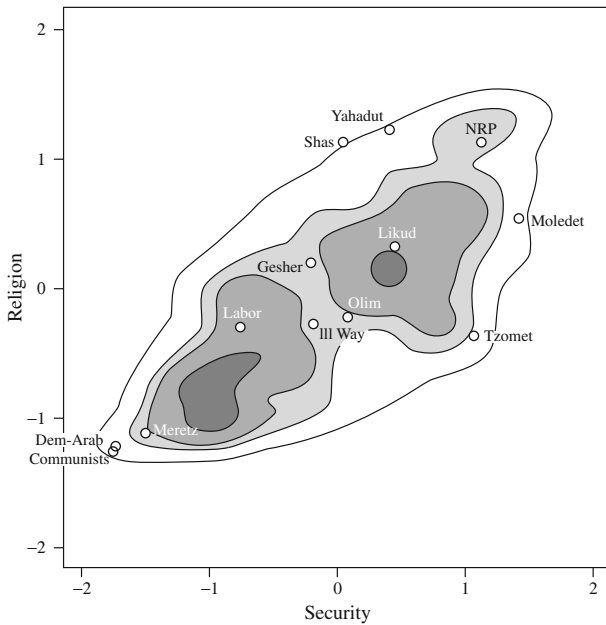
Fig. 4 Party positions and voter distribution in Israel in 1996

with other smaller parties. In the recent election of February 2009, the smaller parties, estimated to be located in the upper right quadrant of the factor space gained some further electoral support. Netanyahu, the leader of Likud, then constructed a winning coalition with the support of Labor and Israel Beitenu.

Our analysis suggests the following:

(i) In the US polity with a pronounced majoritarian electoral system, the two parties, or their candidates, adopt divergent positions that are symmetrically opposed.[30]

(ii) In Israel with a very proportional electoral system (but with a 2% cut-off) the major parties typically adopt positions relatively close to the electoral origin, while the smaller parties occupy quite divergent positions. The result of bargaining between a major party and smaller parties will tend to result in centrist outcomes.[31] The difficulty facing such a polity, especially when the political configuration is fragmented, is that agreement between the parties may be difficult to attain. We comment in the conclusion on other empirical work that has a bearing on this observation

Thus, a very rough interpretation of the significance of policy change, when normalized with respect to a natural characteristic of the electoral distribution, suggests

---

[30] In particular, an estimate of policy changes from one administration to another will be of the order of one esd.

[31] Policy changes from one government to another can be expected to be of the order of at most one esd. If the centrist Kadima party is included in the coalition, then policy switches will be less.

that volatility, in terms of change in the political outcome, is a function of the nature of the electoral system. In this article, we present a general electoral model to provide a formal account of this variation in political configuration.

3 Valence in the electoral model

The model we present is intended to cover both elections between *candidates* for office, such as president, from differing parties, as well as elections involving leaders of different parties. We shall use the term political leaders for both cases.

We assume first that leaders adopt positions to maximize their vote share in the context of a stochastic electoral model. Each leader, j is characterized by an *intrinsic (or exogenous) valence*, denoted by Λ_j.[32] This can be estimated as the intercept term in the pure spatial model, and can be interpreted as the average electorally perceived quality of each leader. Exogenous valence is estimated with respect to a baseline leader, so in the 2008 US election model we set $\Lambda_{\text{Obama}} = 0$, and estimate Λ_{McCain}. We use the notion of a *convergence* coefficient, presented in Schofield (2007a) to show that the difference between Λ_{Obama} and Λ_{McCain} is sufficiently small so that the equilibrium of the pure spatial model is one where the two candidates adopt identical positions at the electoral origin.

The notion of exogenous valence is then extended to include heterogenous, *sociodemographic* valence terms. These sociodemographic valences cause each party to seek out any group in the electorate which has a propensity to favor that leader independently of the leader's declared policy position. In our simulation of the joint sociodemographic model, we found that even with these sociodemographic valences the equilibrium positions in the 2008 US election were not perturbed from the electoral origin.

We then extended the model with exogenous and sociodemographic valences, by including the electoral perceptions of the candidate traits. These candidate traits add considerably to the significance of the valence model. Since these perceptions are individually based, and, therefore, determined by voter position, we can use simulation techniques to compute the equilibrium positions implied by the full valence model. We found that the local equilibrium of the full model with traits was one where candidates adopted positions slightly different positions at $z_{\text{Obama}} = (0.10, -0.07)$ and $z_{\text{McCain}} = (0.13, -0.12)$.

To account for the disparity between the simulated local equilibrium positions of the candidates and their estimated positions, we included *activist* valence in the formal model. We may regard activist valence as a kind of *endogenous* valence since it is the consequence of bargaining between party and activists.[33] A party that has in the past tended to adopt a policy position that favors a particular group may also benefit from the provision of resources, such as money and time, from activists belonging the group. The possibility of obtaining such resources, to enhance the electoral success of

---

[32] See for example the formal model in Serra (2010) and an empirical model of the 2008 election in Jessee (2010).

[33] See the models in Grossman and Helpman (1994, 1996, 2001) and Baron (1994).

a candidate, will exert a centrifugal force, drawing the candidate closer to the group. The marginal calculation by each candidate can be interpreted as a *balance condition*, which incorporates all the valence terms. The term balance is used because it involves equating the opposed centripetal attraction of the electoral center and the centrifugal activist force.

The empirical work presented here for the United States suggests that the endogenous valence terms for the candidates are very similar, so that, for the pure spatial model, the centripetal electoral force should dominate. Because the candidates diverge from the center, we infer that the centrifugal activist valence terms are very significant. We argue that since the electoral system is highly majoritarian, potential activist groups will tend to coalesce, so as to increase their influence on their chosen party. As we noted in the Sect. 1, increasing campaign expenditure by parties in the United States reflects the increase of activist influence.

Under proportional representation, as in Israel, major parties will be characterized by high exogenous valence, in comparison to the more peripheral, low valence parties. The balance condition will cause the high valence parties to pay less heed to sociodemographic valence, and they will be less dependent on activist valence. As a consequence, they will tend to be located near the electoral center. In contrast, small parties will tend to represent the interests of very specific groups in the society. Their exogenous valence, which is a measure of the perceived quality of the party leader in the whole electorate, will be very low. The centripetal electoral effect on small parties will be dominated by the centrifugal effect, and they will tend to adopt positions far from the electoral center.

In the application of the model to Israel in 1996, we show the intrinsic valence of such small parties is indeed very low. This is a general phenomenon, which holds true for models of elections in Poland and Turkey, as well as Israel.[34] The vote share of such parties will be very dependent on sociodemographic valence, as well as on the support of specific activist groups. Thus, the centrifugal force on such parties will be further enhanced.

As illustrated by the case of Israel, relatively small parties will be pivotal for the formation of coalition government. Such parties may expect to gain office, and bring important policy rewards to their activist supporters. Thus, these specific sociodemographic groups will, in expectation, gain from the support they provide to these parties. Both parties and activist groups will be motivated to maintain this mutually beneficial arrangement, and the leaders of such groups will have little motivation to coalesce with other groups. The high level of political fragmentation will be maintained unless a dominant center party can attract some of the relatively radical activists.

To develop this argument, we first consider the United States. Section 4.1 presents a standard binomial logit model for the 2008 presidential election. This model does not involve candidate positions, so we then develop a spatial mixed logit model that does involve candidate positions. Section 4.2 obtains the conditions that characterize the local Nash equilibria in the models with exogenous and sociodemographic valences

---

[34] See Schofield et al. (2010b) for Poland. Schofield et al. (2011) for Turkey.

as well as voter perceptions of the candidate traits. We then use these formal models to estimate the activist influences for this election on the candidates.

In Sect. 5, we perform the same analysis for the 1996 election in Israel, making use of the earlier analysis of Schofield and Sened (2006). Although we do have have available the voter perceptions of character traits of the party leaders, we find that the estimates of exogenous valence obtained from a joint spatial model are sufficiently different to account for the fact that parties do not converge to the electoral origin. We briefly comment on the recent election of 2009, and suggest that the loss of dominance by the centrist party Kadima was due to the decrease of sociodemographic and activist valence by the Labor party.

4 The election of 2008 in the United States

4.1 Empirical analysis

The 2008 American National Election Study (ANES 2008) introduced many new questions on political issues in addition to the existing set. Assignment of respondents into the "new" or "old" set was random, with 1,059 respondents assigned to the "new" condition and having completed the follow-up post-election interview.

The post-election interviews asked respondents whom they voted for, if at all. Since we use a conditional logit model, which requires data for both respondents and candidates (which we only have for the major party candidates) we removed observations where respondents claimed to have voted for a presidential candidate other than McCain or Obama, or not to have voted at all.

To create the two-dimensional policy space, 23 survey items were selected to broadly represent the economic and social policy dimensions of American political ideology (see Appendix 2 for question wording). There were multiple questions for abortion, gay and African American issues. These three sets of questions were combined using factor analysis to give three separate scales.

Factor analysis of the survey was then used to obtain measures of individual locations in the policy space (see Table 1 for factor loadings).

The ANES also includes questions on seven qualities or traits associated with Obama and McCain, asking respondents about the traits of the candidates, including the terms "moral, caring, knowledgable, strong, dishonest, intelligent, out of touch." Factor analysis of these questions gave two factors, and the resulting factor scores were used as estimates of voter perceptions of the candidate's personal traits.

To calculate the presidential candidate positions, we took advantage of new survey questions which asked respondents to locate the positions of Obama and McCain on seven distinct issues.

These seven questions (government spending, universal health care, citizenship for immigrants, abortion when non-fatal, abortion when gender incorrect, aid to blacks, and liberal-conservative) were otherwise worded the same as the corresponding items from the 23 policy issue questions.

To find McCain's ideal point, we simply took the average response for each of his seven candidate location questions. We then repeated the process using Obama's can-

Table 1 Factor loadings for economic and social policy

Question	Economic policy	Social policy
1. Government services	0.53	0.12
2. Universal health care	0.51	0.22
4. Government bigger	0.50	0.14
5. Government or market	0.56	
9. Welfare spending	0.24	
6. Less government	0.65	
7. Equality	0.14	0.37
8. Tax companies	0.28	0.10
12. Abortion scale		0.55
11. Immigrant scale	0.12	0.25
13–16. Gay scale		0.60
17. Traditional values		0.53
18. Gun access	0.36	
19–22. Afr. Amer. scale	0.14	0.45
23. Liberal v conservative	0.30	0.60
Eigenvalue	1.93	1.83

Table 2 Descriptive data

	Econ	Policy		Social	Policy		n
	Mean	s.e.	95% C.I	Mean	s.e	95% C.I	
Activists							
Democrats	−0.20	0.09	[−0.38, −0.02]	1.14	0.11	[0.92,1.37]	80
Republicans	1.41	0.13	[1.66, 1.16]	−0.82	0.09	[−0.99, −0.65]	40
Non-activists							
Democrats	−0.17	0.03	[−0.24, −0.11]	0.36	0.04	[0.29,0.44]	449
Republicans	0.72	0.06	[0.60,0.84]	−0.56	0.05	[−0.65, −0.46]	219
							788

didate location questions. See Tables 2 and 3 for the descriptive data and the estimated positions of the two candidates.

Respondents were coded as activists if they claimed to have donated money to a candidate or party. The survey data gave information on whether the respondent was African American, Hispanic, female, working class, from the South. Additional data on age, number of years of education, and level of income were used to construct eight different sociodemographic variables.

Figure 1, above, gave the voter distribution, while Fig. 2 gave the activist distribution.

Table 3 Obama and McCain perceived positions

Question	Obama	McCain
Estimated position on economic policy	−0.22	0.59
Estimated position on social policy	0.75	−0.37

As noted above, the positions of the major presidential candidates, McCain and Obama, in 2008 were estimated using the perceptions of the sampled individuals. These positions were:

$z_{\text{Obama}} = (x_{\text{Obama}}, y_{\text{Obama}}) = (-0.22, 0.75),$

$z_{\text{McCain}} = (x_{\text{McCain}}, y_{\text{McCain}}) = (0.59, -0.37).$

We now use the formal model to analyze this election.

4.2 Estimation of political equilibria

Obama's victory on November 4, 2008 suggests that it was the result of an overall shift in the relative valences of the Democrat and Republican candidates from the election of 2004. In fact, since Obama took 52.3% of the vote, a simple estimate of the probability, ρ_{obama}, of voting for Obama is given by

$$\rho_{\text{Obama}} = [0.523] = \frac{\exp[\Lambda_{\text{Obama}}]}{1 + \exp[\Lambda_{\text{Obama}}]}$$

It immediately follows that an estimate of Λ_{Obama} relative to Λ_{McCain} is given by

$$\log_e \left[\frac{0.523}{0.477} \right] = \log_e[1.096]$$

$$\simeq 0.09.$$

In fact there were differential shifts in different regions of the country. In a region of the country from West Virginia through Tennessee, Arkansas, and Oklahoma, there was a shift of 20% in the increase in the republican vote, suggesting a change of about 0.6 in McCain's valence advantage.

To model this election, we first constructed a *pure positional binomial logit model*.

According to this positional model, a voter i, with preferred position (x_i, y_i) is estimated to vote Republican with probability

$$\rho_{\text{rep}} = \frac{\exp(\Lambda_r + bx_i + cy_i)}{1 + \exp(\Lambda_r + bx_i + cy_i)}. \tag{1}$$

We estimated these coefficients to be $(\Lambda_r, b, c) = (-0.74, 1.49, -1.80)$, with standard errors $(0.11, 0.13, 0.15)$, respectively. All were significant at the 0.001 level.

This cleavage line derived from this the equation gives the locus of voting with equal probability for one or other of the candidates. This cleavage line is given by the equation

$$y = 0.82x - 0.4. \tag{2}$$

This cleavage line misses the origin, and goes through the point $(0, -0.4)$, indicating the valence advantage of Obama. The coefficient Λ_r is a measure of the (negative) relative valance of McCain with respect to Obama for this positional model. This cleavage line is similar to those obtained by Schofield et al. (2003) for the presidential elections of 1964 and 1980. (One difference between this earlier estimate and the one presented here was that in 1980 they found that Reagan had a valence advantage over Carter.)

These positional models do not explicitly involve the candidate positions, and so cannot be used to determine political equilibria. We now discuss the spatial models, presented in Table 4.

The electoral covariance matrix for the sample is given by

$$\nabla_0 = \begin{bmatrix} 0.80 & -0.127 \\ -0.127 & 0.83 \end{bmatrix}.$$

The principal component of the electoral distribution is given by the vector $(1.0, -1.8)$ with variance 1.02, while the minor component is given by the orthogonal eigenvector $(1.8, 1.0)$ with variance 0.61.

Model (1) in Table 4 shows the coefficients in 2008 for the pure spatial model to be

$$(\Lambda_{\text{Obama}}, \Lambda_{\text{McCain}}, \beta) = (0, -0.84, 0.85).$$

Table 4 indicates, the loglikelihood, Akaike information criterion (AIC), and Bayesian information criterion (BIC) are all quite acceptable, and all coefficients are significant with $P < 0.01$.

Note that these parameters are estimated when the candidates are located at the estimated positions. Again, Λ_{McCain} is the relative negative exogenous valence of McCain, with respect to Obama, according to the model $\mathbb{M}(\Lambda, \beta)$. We assume that the parameters of the model remain close to these values as we modify the candidates positions in order to determine the equilibria of the model.

According to the model $\mathbb{M}(\lambda, \beta)$, the probability that a voter chooses McCain, when the McCain and Obama positions are at the electoral origin, $\mathbf{z}_0 = ((0, 0), (0, 0))$ is

$\rho_{\text{McCain}} = [1 + \exp(0.84)]^{-1} = [1 + 2.31)]^{-1} = 0.3$.

Then $\beta(1 - 2\rho_{\text{McCain}}) = 0.85 \times 0.4 = 0.34$.

The characteristic matrix (essentially the Hessian of McCain's vote function at $\mathbf{z}_0$ is:

$$\begin{aligned} C_{\text{McCain}} &= [2\beta(1 - 2\rho_{\text{McCain}})\nabla_0 = [2 \times 0.34 \times \nabla_0] - I = (0.68)\nabla_0 - I \\ &= (0.68)\begin{bmatrix} 0.8 & -0.127 \\ -0.127 & 0.83 \end{bmatrix} - I = \begin{bmatrix} 0.54 & -0.086 \\ -0.086 & 0.56 \end{bmatrix} - I \\ &= \begin{bmatrix} -0.46 & -0.086 \\ -0.086 & -0.44 \end{bmatrix} \end{aligned}$$

Table 4 β-Spatial conditional logit models for USA 2008

	Spatial	Sp. & Traits	Sp. & Demog	Full
McCain valence Λ	-0.84^{***} (0.11)	-1.08^{***} (0.13)	-2.60^{**} (0.93)	-3.58^{***} (1.05)
Distance β	0.85^{***} (0.06)	0.78^{***} (0.07)	0.86^{***} (0.07)	0.83^{***} (0.08)
McCain traits		1.30^{***} (0.17)		1.36^{***} (0.19)
Obama traits		-1.02^{***} (0.15)		-1.16^{***} (0.18)
Age			-0.01 (0.01)	-0.01 (0.01)
Female			0.29 (0.23)	0.44 (0.26)
African American			-4.16^{***} (1.10)	-3.79^{***} (1.23)
Hispanic			-0.55 (0.41)	-0.23 (0.45)
Education			0.15^{*} (0.06)	0.22^{***} (0.06)
Income			0.03 (0.02)	0.01 (0.02)
Working class			-0.54^{*} (0.24)	-0.70^{**} (0.27)
South			0.36 (0.24)	-0.02 (0.27)
Observations	788			
Log likelihood (LL)	-298.63	-243.14	-250.25	-206.88
AIC	601.27	494.28	520.50	437.77
BIC	610.59	512.92	567.11	493.69

Standard errors in parentheses. $^{*}P < 0.05$; $^{**}P < 0.01$; $^{***}P < 0.001$
Vote for Obama is the baseline outcome

The "convergence coefficient" is

$$c = 2\beta(1 - 2\rho_{\text{McCain}})trace\nabla_0 = 2(0.34)(1.63) = 1.1.$$

Schofield (2007a) shows that the necessary condition for convergence to $\mathbf{z}_0$ is that $c < 1$. Note that c is dimensionless, and therefore independent of the units of measurement.

The estimate for c exceeds this critical value for convergence. However, the determinant of C_{McCain} is positive and trace is negative so both the eigenvalues of C_{McCain} are negative. Standard results of calculus show that the origin is a maximum of McCain's vote share function. Simulation of the pure spatial model confirmed that $\mathbf{z}_0$ was an LNE. Indeed it was shown to be a Pure Strategy Nash equilibrium (PNE).

We also considered a spatial model where the x and y axes had different coefficients, $\beta_1 = 0.8$, $\beta_2 = 0.92$. The analysis showed the the Hessian for this case had negative eigenvalues, so again $\mathbf{z}_0$ is a LNE. This model is essentially the same as the model with a single β.

We now turn to the models with traits and sociodemographics.

Table 4, above, gave the various spatial models with these additional valences.

We found that the loglikelihoods of the pure sociodemographic model and pure traits models to be to be -427 and -356, respectively. Comparison of the loglikelihoods for the pure spatial model and the model with traits, as given in Table 4 shows that the perception of character traits is important for the statistical significance

Table 5 Comparison of LL for US spatial models in 2008

	JST	ST	S	T
JST	na	36	92	150
ST	−7	na	55	114
S	−92	−55	na	58
T	−150	−114	−58	na

JST Joint spatial with traits, *ST* spatial with traits, *S* pure spatial, *T* Pure traits

of the model. As Table 5 shows, the difference in the loglikelihoods of the spatial model with traits and the pure traits model is $-243 + 357 = 114$, while the difference between the full spatial model with traits and sociodemographics against the traits model is $-206 + 357 = 150$.

Simulation of the full spatial model with traits and sociodemographics showed that the LNE (and PNE) was one where the candidates adopted the positions $z_{Obama} = (+0.10, -0.07)$ and $z_{McCain} = (+0.13, -0.12)$.

We can, therefore, write

$$\mathbf{z}^{el} = (z_{Obama}^{el}, z_{McCain}^{el}) = ((+0.10, -0.07), (+0.13, -0.12))$$

since the joint model with traits has no activist valence terms.

This equilibrium is only a slight perturbation from the joint origin. We can infer that though the traits add to the statistical significance of the stochastic model they do not significantly affect the equilibrium. Analysis of the relationship between perceptions of candidate traits and vote choice showed that there were weak correlations and these had only a slight effect on the strong convergence induced by the electoral pull.

The results of the Appendix 1 show that $\mathbf{z}^{el}$ can be interpreted as the vector of "weighted electoral means" in a full model with activists. Assuming that the estimated candidate positions, $\mathbf{z}^*$, are in equilibrium with respect to the activist model, then by the balance condition, we obtain:

$$\mathbf{z}^* - \mathbf{z}^{el} = \begin{bmatrix} & McCain & Obama \\ x & 0.59 & -0.22 \\ y & -0.37 & +0.75 \end{bmatrix} - \begin{bmatrix} & McCain & Obama \\ x & +0.13 & +0.10 \\ y & -0.12 & -0.07 \end{bmatrix}$$

$$= \frac{1}{2\beta} \frac{d\mu}{d\mathbf{z}}(\mathbf{z}) = \begin{bmatrix} & McCain & Obama \\ x & 0.46 & -0.32 \\ y & -0.25 & 0.82 \end{bmatrix}.$$

Here

$$\frac{d\mu}{d\mathbf{z}}(\mathbf{z}) = \left(\frac{d\mu_{mc}}{dz_{mc}}(z_{mc}), \frac{d\mu_{ob}}{dz_{ob}}(z_{ob}) \right)$$

is the pair of direction gradients, induced by activist preferences, acting on the two candidates. The difference between $\mathbf{z}^*$ and $\mathbf{z}^{el}$ thus provides an estimate of the activist pull on the two candidates. In this election, we estimate that activists pull the two candidates into opposed quadrants of the policy space. The estimated distributions

of activist positions for the two parties, in these two opposed quadrants (as given in Fig. 1) are compatible with this inference. The means of these activist positions are:

$$\begin{bmatrix} & \text{Rep Act} & \text{Dem Act} \\ x & 1.41 & -0.2 \\ y & -0.82 & 1.14 \end{bmatrix}.$$

Miller and Schofield (2003, 2008) propose a model where activists have eccentric or ellipsoidal utility functions. If we assume that the Democrat activists tend to be more concerned with social policy and Republican activists tend to be more concerned with economic policy, then we have an explanation for the candidate shifts from the estimated equilibrium. Note in particular that the distribution of activist positions for the two parties, given in Fig. 2, looks very different from the voter positions, given in Fig. 1. The latter is much more heavily concentrated near the electoral origin, while the former tends to be dispersed.

Miller and Schofield (2008) also emphasized the potential conflict between economically conservative and socially conservative Republican activists. In Indiana in February 2010, the incumbent Democrat Senator, Evan Bayh, announced that he would retire. This set off a contest by local "tea party" social conservatives against the Republican National Committee's support for Dan Coats, an economic conservative contender for the Senate seat. This example just illustrates the degree to which contenders for political office require support from activist groups with very different agendas.

When the candidates are at their estimated positions, the estimated vote shares, according to the traits model, are $(V_{\text{Obama}}, V_{\text{McCain}}) = (0.68, 0.32)$. Since the actual vote shares are $(0.52, 0.48)$, it appears that the trait model may give a statistically plausible account of voter choice, but it does not provide, by itself, a good model of how candidates obtain votes. We suggest that the missing characteristic of this model of the election is the effect on the vote by the contributions of party activists.

Indeed, we suggest that the addition of activists to the model can account for the difference between convergent, equilibrium positions and the divergent, estimated candidate positions, as obtained by Enelow and Hinich (1989) and Poole and Rosenthal (1984), respectively, in their various analyses of U.S. elections.

As we noted above, we could also interpret $\frac{d\mu}{d\mathbf{z}}(\mathbf{z})$ as the gradient obtained from a model where candidates have policy preferences derived from utility functions $(\mu_{\text{mc}}, \mu_{\text{ob}})$. Duggan and Fey (2005) have explored such a model for the case of a deterministic vote model, and obtained symmetry conditions for equilibrium similar to those obtained earlier by McKelvey and Schofield (1987). However, in such a model of policy seeking candidates, a candidate must be willing to adopt a losing position because of strong preferences for particular policies.

It is possible that our estimates of the positions, $z_{\text{Obama}} = (-0.22, 0.75)$ and $z_{\text{McCain}} = (0.59, -0.37)$, are incorrect. However, these estimated positions give us a statistically significant model of voter choice. We argue that the most plausible account for the difference in the estimated and equilibrium positions of the two candidates is the nature of activist competition.[35]

---

[35] See Schofield et al. (2010a) for the US election in 2000.

Table 6 Seats in the Knesset

Party	1988	1992	1996	1999	2003	2006	2009
Left (ADL, Arab, Hadash)	14	5	9	10	9	10	11
Meretz		12	9	10	6	5	3
Labor	39	44	34	28	21	19	13
Center (Olim, Gesher, Shinui)	2	8	11	18	15	7	–
Center (Kadima)						29	28
Likud	40	32	30	19	40	12	27
Shas, Yahadut	15	10	14	22	16	12+6	11+5
NRP, Mafdal	5	6	9	5	6	9	4+3
Moledat, Techiya, Beiteinu	5	3	2	8	7	11	15
Total	120	120	120	120	120	120	120

Note also that this model can be applied to the determination of policy positions of members of the House and Senate of the United States. In particular, we would expect local activist groups to be very heterogenous across states and House constituencies. As a result, policy positions of members of Congress can be expected to be very heterogenous, even within parties.

5 Elections in Israel

Schofield and Sened (2006) estimated various multinomial conditional logit models for the elections of 1988, 1992, and 1996 in Israel.[36] Table 6 gives the election results for 1988–2009, while Fig. 4, presented above, showed the electoral distribution in 1996, together with estimates of the party positions. Using the formal analysis, we can readily show that the *convergence coefficient* of the pure spatial model, $\mathbb{M}(\mathbf{\Lambda}, \beta)$ for 1996 greatly exceeds 2 (the dimension of the policy space). Indeed, one of the eigenvalues of the Hessian of the one of the low valence parties, Shas, can be shown to be positive. The principal electoral axis (or principal component of the electoral distribution) can be seen to be aligned at approximately 45° to the security axis. As we now show, this axis is the eigenspace of the positive eigenvalue. It follows from the computation of eigenvalues that low valence parties should position themselves close to this principal axis, as illustrated in the simulation of the model, given in Fig. 5.

The MNL estimation given in Table 7 presents the relative valences in the pure spatial model with respect to Meretz. The table shows that in 1996 Shas had a relative valence of $\Lambda_{\text{Shas}} = -2.02$, while Labor had the highest relative valence of 0.99, with Likud having a valence of 0.78. The spatial coefficient was $\beta = 1.21$, so to use the convergence theorem, we note that the valence difference between Shas and
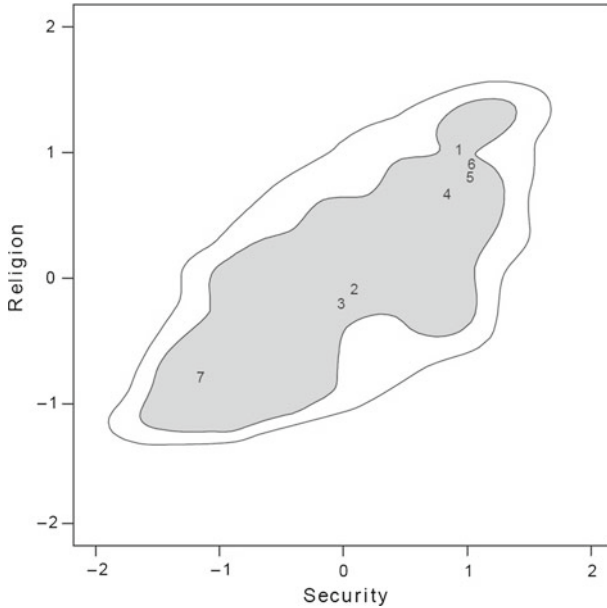

Fig. 5 Estimated local equilibrium positions in the Knesset in 1996. Key: *1* Shas, *2* Likud, *3* Labor, *4* NRP, *5* Moledat, *6* Third way, *7* Meretz

Labor was $0.99 - (-2.02) = 3.01$, while the difference between Shas and Likud was $0.78 - (-2.02) = 2.8$. The electoral covariance matrix is

$$\nabla_0 = \begin{bmatrix} 1.0 & 0.591 \\ 0.591 & 0.732 \end{bmatrix}$$

with trace $\sigma^2 = 1.732$. The principal component of this electoral distribution is given by the vector $(1.0, 0.80)$ with variance 1.47, while the minor component is given by $(1.0, -1.25)$ with variance 0.26. We can compute the characteristic matrix of Shas at the origin and the convergence coefficient as follows:

$$\rho_{Shas} \simeq \frac{1}{1 + e^3 + e^{2.8} + e^{1.4} + e^{0.8}}$$
$$\simeq 0.023.$$
$$2\beta(1 - 2\rho_{Shas}) = 2 \times 1.21 \times 0.95 = 2.30$$
$$\text{so } C_{Shas} = (2.3)\nabla_0 - I$$
$$= \begin{bmatrix} 1.3 & 1.36 \\ 1.36 & 0.69 \end{bmatrix}.$$
$$\text{and } c = 2.3 \times 1.732 = 3.98.$$

From the estimate of C_{Shas}, it follows that the two eigenvalues are 2.39 and -0.39, giving a *saddlepoint*, and a value of 3.98 for the convergence coefficient. This exceeds

Table 7 Spatial model of the Israel election 1996, wrt Meretz

Variable	Party	Coefficient	Lower 95% bound	Upper 95% bound
β Spatial		1.207***	1.076	1.338
Λ_Valence	Likud	0.777***	0.400	1.154
	Labor	0.990***	0.663	1.316
	NRP	−0.626***	−1.121	−0.132
	Moledat	−1.259***	−1.858	−0.660
	Third way	−2.291***	−2.841	−1.741
	Shas	−2.023***	−2.655	−1.392
Convergence c		3.98	3.70	4.26
LL = − 777.0	$n = 922$	***$P < 0.001$		

LL Log likelihood

Table 8 Comparison of LL for Israel models for 1996

	$\mathbb{M}_2$	Joint	Spatial	Socio-Dem.
	Joint	na	82	249
$\mathbb{M}_1$	Spatial	−82	na	167
	Socio-Dem.	−249	−167	na

the necessary upper bound of 2. The estimate for the standard error on ρ_{Shas} is 0.008, so the 95% confidence interval is [0.007, 0.02]. Note that this interval includes the actual sample vote share of 2% for Shas. The standard error on β is 0.065 so the standard error on c is of order 0.14, and we can infer that, with high probability, the convergence coefficient exceeds 2.0.

Using the above estimate for the major eigenvalue, we find that the major eigenvector for Shas is (1.0, 0.79), and along this axis the Shas vote share function increases as the party moves away from the origin. The minor, perpendicular axis associated with the negative eigenvalue is given by the vector $(1, -1.26)$. Any LNE for the model $\mathbb{M}(\Lambda, \beta)$ will be one where all parties are located on the major eigenvector.

We also constructed a joint MNL model, $\mathbb{M}(\Lambda, \theta, \beta)$, and a pure sociodemographic model of the election, $\mathbb{M}(\Lambda, \theta)$, details of which can be found in Schofield and Sened (2006). Table 8 reports the differences in the log likelihoods of the various models.

Figure 5 gives one of the local Nash equilibria, obtained by simulation of the model. Since this model does not involve activist terms, we can infer that this equilibrium gives an estimate of the weighted electoral means, $\mathbf{z}^{el}$, for the parties: This vector, $\mathbf{z}^{el}$, is given by:

$$\begin{bmatrix} \text{Party} & \text{Meretz} & \text{Moledat} & \text{IIIWay} & \text{Labor} & \text{Likud} & \text{NRP} & \text{Shas} \\ x & -1.1 & 1.0 & 1.0 & 0.0 & 0.2 & 0.9 & 1.0 \\ y & -0.8 & 0.8 & 0.8 & -0.2 & 0.0 & 0.6 & 1.0 \end{bmatrix}$$

All these equilibrium positions lie very close to an eigenvector (1.0, 0.85). It thus appears that the only effect of the inclusion of the sociodemographic variables is to

slightly rotate the principal eigenvector in an anticlockwise direction. In all, five different LNE were located. However, in every equilibrium, the two high valence parties, Labor and Likud, were located close to the simulated equilibrium positions shown in Fig. 5. The only difference between the various equilibria were slight differences in the positions of Shas, NRP, and Moledat.

It is evident that if the high valence party occupies the electoral origin, then each party with low valence can compute that its vote share will increase by moving up or down the principal electoral axis. In seeking local maxima of the vote shares, all parties other than the highest valence party should vacate the electoral center. Then, however, the first-order condition for the high valence party to occupy the electoral center would not be satisfied. Even though this party's vote share will be little affected by the other parties, it too should move from the center. The simulation for 1996 is compatible with the formal analysis: low valence parties, such as the NRP and Shas, in order to maximize vote shares must move far from the electoral center. As with the pure spatial model, their optimal positions will lie either in the "northeast" quadrant or the "south-west" quadrant. The vote-maximizing model, without any additional information, cannot determine which way the low valence parties should move.

The equilibrium position of Shas, by the joint model, will give greater weight to those voters who are observant. As Fig. 4 makes clear, Shas, Moledat, and NRP are located in the upper quadrant of the policy space. On the other hand, since the valence difference between Labor and Likud was relatively low, their local equilibrium positions will be close to, but not identical to, the electoral mean. Intuitively, it is clear that once the low valence parties vacate the origin, then high valence parties, like Likud and Labor, should position themselves almost symmetrically about the origin, and close to the principal axis.

We now compare the LNE obtained from the joint model with the vector, $\mathbf{z}^*$, of estimated positions given in Fig. 4:

$$\begin{bmatrix} \text{Party} & \text{Meretz} & \text{Moledat} & \text{IIIWay} & \text{Labor} & \text{Likud} & \text{NRP} & \text{Shas} \\ x & -1.5 & 1.4 & -0.2 & -0.8 & 0.6 & 1.0 & 0.0 \\ y & -1.0 & 0.5 & -0.4 & -0.2 & 0.2 & 1.1 & 1.1 \end{bmatrix}.$$

We hypothesize that $\mathbf{z}^*$ is a local equilibrium of the full activist model: the difference, $\mathbf{z}^* - \mathbf{z}^{\text{el}}$, between the vector of positions and the equilibrium of Fig. 5 is of order

$$\begin{bmatrix} \text{Party} & \text{Meretz} & \text{Moledat} & \text{IIIWay} & \text{Labor} & \text{Likud} & \text{NRP} & \text{Shas} \\ x & -0.4 & 0.4 & -1.2 & -0.8 & 0.4 & 0.1 & -1.0 \\ y & -0.2 & -0.3 & -1.2 & 0.0 & 0.2 & 0.5 & 0.1 \end{bmatrix}.$$

Thus, this vector gives an estimate of the influence of activist groups on the parties:

$$\mathbf{z}^* - \mathbf{z}^{\text{el}} = \frac{1}{2\beta} \left[\frac{d\mu_1}{dz_1}, \ldots, \frac{d\mu_p}{dz_p} \right].$$

Schofield and Sened estimate $\beta = 1.117$ for the joint model, so we obtain

$$\left[\frac{d\mu_1}{dz_1}, \ldots, \frac{d\mu_p}{dz_p}\right] = 2\beta(\mathbf{z}^* - \mathbf{z}^{el})$$

$$= \begin{bmatrix} \text{Party} & \text{Meretz} & \text{Moledat} & \text{IIIWay} & \text{Labor} & \text{Likud} & \text{NRP} & \text{Shas} \\ x & -0.9 & 0.9 & -2.7 & -1.78 & 0.9 & 0.22 & -2.2 \\ y & -0.45 & -0.67 & -2.68 & 0.0 & 0.45 & 1.12 & 0.22 \end{bmatrix}$$

Although we have not performed the empirical analysis for the elections of 2003 and 2006, we can expect a similar result to hold. The analysis given in Schofield and Sened (2006) for the elections of 1992 and 1988 shows that in 1988 the two eigenvalues for Shas were $+2.0$ and -0.83, while in 1992 the eigenvalues for this party were $+2.12$ and -0.52. Just as in 1996, the theoretical model of vote maximization implies that all parties should be located on a principal electoral axis. The positioning of Shas off the principal electoral axis enables it to pivot between the two major parties, in the sense that it tended to be crucial for the formation of winning coalitions.

As Table 7 shows, after the elections of 1996, 1999, and 2003 any winning coalition based on either Labor or Likud needed additional support of Shas. In 1996, Netanyahu of Likud formed a government with Shas, but after Likud lost seats in 1999, it was the turn of Barak of Labor to form a government, again with Shas, followed in 2001 by Likud, led by Sharon, with Shas. In consequence, even though Shas controlled few seats in this period, it had significant bargaining power.

5.1 The elections of 2006 and 2009

This pattern of coalition government was transformed, to some degree, when Amir Peretz stood against Shimon Peres and won the election for leadership of Labor in November 2005.

Sharon then left the Likud Party and allied with Peres and other senior Labor Party members, to form the new party, *Kadima* ("Forward"). We can infer that the coalition of Sharon and Peres positioned Kadima at the center of the policy space. Because of Sharon's stroke in January 2006, Ehud Olmert took over as leader of Kadima, and in the election of March 2006, the new party was able to take 29 seats, while Likud only took 19 seats. One surprise of the election was the appearance of a Pensioners' party with 7 seats. A possible coalition of Likud and the religious parties, opposed to Kadima, did not have the required 61 seats for a majority (even with the Pensioners' Party). Schofield (2007b) discussed this election and argued that Kadima was at the core position, since no majority coalition could agree to overturn the Kadima position.[37] However, this "core property" was unstable, in the sense that it could be destroyed by small changes in positions or strengths of the parties.

---

[37] For a discussion of the core see Laver and Schofield (1990). For a spatial voting game in a legislature, the core is given by the intersection of legislative median lines between pairs of parties that pivot. If these lines do not intersect then the core is empty. In this case, the set bounded by these median lines is called the "heart". By definition, when the core is empty, then the heart is non empty. See Schofield (1999, 2007b)

As a result, Olmert needed the support of Labor to be able to deal with the complex issue of fixing a permanent border for Israel. The debacle in Lebanon severely weakened Olmert's popularity, and the 61 members of the Kadima-Labor coalition voted to bring Israel Beiteinu into the coalition. The report, in April 2007, on the failure of the government during the war with Lebanon in Summer 2006 seemed to threaten the Kadima-Labor–Israel Beiteinu coalition by bringing about a change in the Labor party leadership. Barak then won the election for the Labor Party leadership on June 12, 2007, and became Minister of Defense in the government on June 18, while Shimon Peres became President. In November 2007, Olmert proposed a land-for-peace proposal, possibly involving the separation of Jerusalem, and on January 15, 2008, Avigdor Lieberman, chairman of Israel Beiteinu announced that the party would quit the government because of disagreement over issues such as Jerusalem and negotiations with Hamas.

On February 3, 2008, Barak agreed to remain in the coalition, thus helping to sustain Kadima in power. However, in August 2008, Olmert faced charges of corruption, and formally resigned as leader of Kadima on September 21. He immediately gave an interview (Olmert 2008) in which he asserted that Israel would have to lose sovereignty over Jerusalem, and would have to come to an agreement with Syria by giving up the Golen Heights in return for Syrian forswearing their connections with Iran, Hezbollah, and Hamas.

The new leader of Kadima, and Prime Minister designate, Tzipi Livni, then had to face a revolt by Shas, over these security issues. On October 26, 2008, she announced, that she had failed to form a viable coalition, and an election would occur in February 2009. Even though the Kadima government was weakened, it responded to rocket attacks by Hamas from Gaza, and launched a 3 week attack on Gaza at the end of December 2008.

In the election of 2009, as Table 6 shows, the Pensioners' Party disappeared, and both Likud and Israel Beiteinu gained seats. Labor lost significantly, presumably because of the loss of valence by its leader, Ehud Barak. The core was destroyed, and it was unclear what government would form. Both Livni and Benjamin Netanyahu, of Likud, claimed the electoral mandate. However, on February 20, Avigdor Lieberman took the role of *formateur* of the coalition game, and offered his support to Netanyahu. On March 24, a majority of the Labor Party central committee voted to support Netanyahu, in return for four cabinet positions, and the retention of the defense portfolio by Barak. Tzipi Livni refused the offer to join this unity coalition government of Likud, Labor, Shas, and Israel Beiteinu, and will be in opposition. As prime minister designate, Netanyahu declared on March 26 that he would negotiate with the Palestinian Authority for peace. Five days later he was sworn in as Prime Minister, after a vote of 69 to 45, with the abstention of five Labor members (one Arab member of the Knesset was absent). Avigdor Lieberman became foreign minister. Although Netanyahu has tended to avoid mention of a sovereign Palestinian state, he declared in December 2009 that in order to proceed with this policy, he was willing to consider inviting Livni to join in a grand coalition.

In March 2010, during Vice President Biden's visit to Israel it was announced that Israel would add 1,600 housing units in eastern Jerusalem. Although the Obama administration was angered by the timing of the announcement, Netanyahu insisted

that Israel would go ahead with the construction. However, President Shimon Peres said: "We cannot afford to unravel the delicate fabric of friendship with the United States. Today we are also at a decisive moment and we must decide without the determination of external parties." However, on May 31,2010, there was an attack by Israeli commandoes against a boat traveling in international waters and carrying humanitarian supplies for Gaza. Nine people in the convoy were killed. The convoy was partly organized by a Turkish organization, Insani Yardim Vakfi. It is unclear who ordered the attack, but Netanyahu immediately cancelled a meeting that had been arranged with Obama.

Eventually, in September 2010, negotiations started in Washington, involving Netanyahu, Mahmoud Abbas (the President of the Palestinian Authority), King Abdullah II of Jordan and President Hosni Mubarak of Egypt.

5.2 Coalitions in Israel

We can see the nature of bargaining over this coalition government by joining the median lines between pairs of parties that pivot between majority coalitions after the 2009 election, as shown in Fig. 6. When these medians do not intersect, then they bound a finite, star shaped set known as the "heart."

Schofield (2007b) argues that the outcomes of coalition bargaining will be a constrained within this set. [38] The complex nature of this set suggests that there are many possible majority coalitions. In particular, small parties such as Shas, Yahadut, and Israel Beiteinu may join in government and may thus influence the outcome of coalition government. We have argued that the positions adopted by the parties are the result of activist choices to support particular parties. Thus, activist groups for these small parties may reason that the party they support has a good chance of taking part in government, thus bringing about policy changes that favor the activists. Consequently, there is little motivation for such activist groups to coalesce. As long as the logic of vote maximization maintains this policy divergence between the parties, then so will activist groups continue to provide support for these small parties. Thus, political fragmentation is preserved

These remarks about recent events in the Knesset are presented to illustrate the great difficulty of maintaining a stable government coalition, even when there is a large, centrally located party, such as Kadima. Such a party should, in principle, be able to dominate bargaining. However, it is only when the center party's leader has high valence is the party able to avoid threats to the government. Without such valence predominance, small parties, and their activist supporters have an incentive to act to maintain political fragmentation.

6 Concluding remarks

This article has argued, on the basis of an equilibrium analysis of elections, that the electoral pull on parties is very different in the United States and Israel, and this is a

---

[38] As mentioned above, when the medians intersect then the heart will collapse to the core, and we may assume that this will be the outcome.
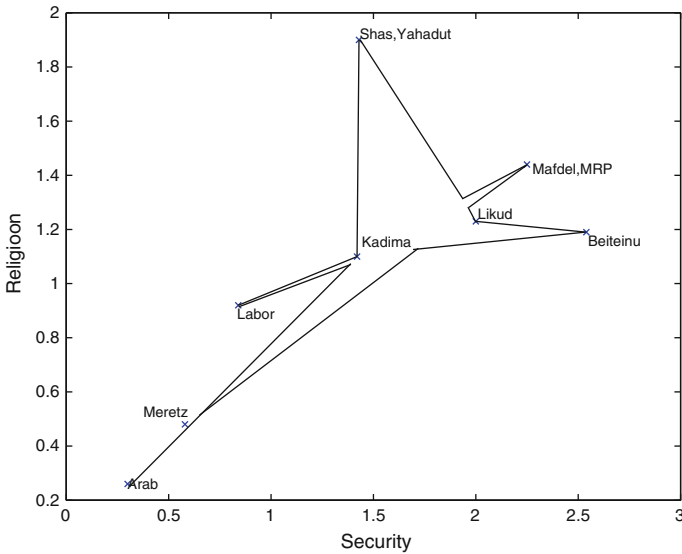
Fig. 6 The heart in Israel in 2008

consequence of the nature of the electoral system and the degree of political fragmentation.

We can express the difference between proportional representation and plurality rule as follows.

Analysis of the equilibria of the spatial valence model of a polity based on *proportional electoral methods*, such as Israel, indicates that only high valence parties will be positioned, in equilibrium, close to the electoral origin. Because of the wide variation in political valence, the low valence parties will move toward the electoral periphery. Activist groups, linked to small parties, may aspire to affect the policy choices of the chosen parties. Bargaining to create winning coalitions occurs *after* the election, and small parties may aspire to membership of government. As a consequence, there need to be no strong tendency forcing activist groups to coalesce, in order to concentrate their influence. If activist groups remain fragmented, then the outcome will be party fragmentation, and this will reinforce the tendency for variation in valence. Empirical analysis of the election in Israel for 1996 showed that the convergence coefficient had a high value of 3.98 in 1996. Other empirical work obtained a value for this convergence coefficient of 6.82 for 1997 in Poland (Schofield et al. 2010b) and of 5.94 for Turkey in 2002 (Schofield et al. 2011).

A standard way of estimating political fragmentation is in terms of the *effective number of party vote strength* (env) or *effective number of party seat strength* (ens).[39] The fragmentation in votes and seats is captured by the fact that in Israel in 1996 both

---

[39] Fragmentation can be identified with the *effective number* (Laakso and Taagepera 1979). That is, let H_v (the Herfindahl index) be the sum of the squares of the relative vote shares and env $= H_v^{-1}$ be the *effective number of party vote strength*. In the same way we can define ens as the effective number of party seat strength using shares of seats.

Table 9 Convergence coefficients (*c*) and fragmentation

	Country		
Variable	US	Britain	Canada
Convergence coeff. (*c*)	[0.40,1.1] (2000–2008)	[0.84, 0.98] (2005–2010)	2.0 (2004)
Political system	Pres. PL	Parl. PL	Parl. PL
env	2.0	3.2 (1997)	4.0 (2004)
env		2.7 (2005)	4.1 (2008)
ens	1.0	2.2 (1997)	3.1 (2004)
ens		2.5 (2005)	3.5 (2008)
	Poland	Turkey	Israel
Convergence coeff. (*c*)	6.82 (1997)	5.94 (2002)	3.98 (1996)
Political system	Frag. PR	Frag. PR, cut off	Frag PR
env	5.5 (1997)	7.7 (1999)	6.5 (1996)
env	7.7 (2005)	4.0 (2007)	10.0 (2009)
ens	3.1 (1997)	5.0 (1999)	6.5 (1996)
ens	5.0 (2005)	2.3 (2007)	10.0 (2009)

Parl parliamentary, *Pres.* presidential, *PL* plurality, *PR* proportional representation, *Frag.* fragmented

env and ens were equal to 6.5. As Table 9 shows, these had risen to 10.0 by 2009. The table shows that the effective numbers for vote shares in Poland and Turkey were also large, in the range [4.0, 7.7]. These three polities all have electoral systems that are approximately proportional, although Turkey introduced an electoral cut-off, which worked to the disadvantage of smaller parties, reducing the ens from 5.0 in 1999 to 2.3 in 2007.

The parliamentary polities of Canada and Britain, based on a plurality electoral system, have convergence coefficients that lie in the range [0.8, 2.0], and the fragmentation measures tend to be much lower than in the PR polities. For Canada, the env is about 4.0, and in Britain it lies in the range [2.7, 3.2]. In the very majoritarian polity of the United States the env is generally about 2.0. The equilibrium analysis of the spatial model presented here indicates that the convergence coefficient had a low value of 1.1 in 2008. In 2000 and 2004, Schofield et al. (2010a,b) estimate the coefficient to be 0.4.

Schofield and Zakharov (2010) suggest that Russia has an electoral system that is in between plurality and proportionality. It has a dominant party, United Russia, supportive of Putin, and the env was only 2.3 in the Duma election of 2007, while the convergence coefficient was estimated to be 1.7.

These comparisons of convergence coefficients are valid because *c* is dimensionless. The higher coefficients in polities with many parties is a result both of higher β-coefficients and greater electoral variance. In contrast, for US presidential elections, the convergence property still holds even when sociodemographic variables and individual perceptions of candidate traits are incorporated into the model. We argue that presidential candidates in the United States are pulled from these convergent equilibrium positions by the influence of interest groups. We suggest that in the election of

2008, social activists dominated in the Democrat party and economic activists in the Republican party. Activist resources are crucial for electoral success. If these interest groups do not coalesce *before* the election, then they will have little impact on political outcomes. Consequently, small parties, or activist groups, such as those led by independent candidates in recent elections, have little expectation of influencing government policy. Their valences will remain low, and they will have little impact, in general, on presidential elections. However, heterogenous local activist groups may influence the policy preferences of members of Congress, and this effect may induce conflict between the President of one party and Senators of the same party, as seems to be the case over the issue of health care in 2009 and 2010. Radical activist groups may also induce conflict within the support coalitions for one or other of the parties, particularly when the party is out of office. This phenomenon can be seen within the Republican party in the aftermath of the 2008 election. We may hypothesize that such conflict is the fundamental cause of realignment in the US polity.

Appendix 1: a formal stochastic model of elections

Details of the pure spatial stochastic electoral model are given in Schofield (2006, 2007a), to which the reader is referred. Here, we extend the presentation given in Schofield et al. (2011) by including electoral perceptions of candidate traits as well as allowing for multiple activist groups.

The aim of the model is to argue that the vote-maximizing equilibrium position of McCain will lie on what we shall call a *balance locus* in the lower right quadrant of the policy space, while Obama's position will lie in the opposite, upper left, quadrant.

The voter utility assumption for the stochastic vote model $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \boldsymbol{\mu}, \beta)$ is:

$$u_{ij}(x_i, z_j) = \Lambda_j + (\theta_j \cdot \eta_i) + (\alpha_j \cdot \tau_i) + \mu_j(z_j) - \beta \|x_i - z_j\|^2 + \varepsilon_j \qquad (3)$$

$$\equiv u_{ij}^*(x_i, z_j) + \varepsilon_j. \qquad (4)$$

Here, $u_{ij}^*(x_i, z_j)$ is the observable component of utility. The set of leaders is $P = \{1, \ldots j, \ldots p\}$. The term Λ_j is the exogenous valence of leader j, relative to the baseline leader. This is estimated from the intercept term of the model. The symbol $\boldsymbol{\theta}$ denotes a set of k-vectors $\{\theta_j : j \epsilon P\}$ representing the effect of the k different sociodemographic parameters[40] on voting for leader j while η_i is a k-vector denoting the ith individual's relevant "sociodemographic" characteristics. The compositions $\{(\theta_j \cdot \eta_i)\}$ are scalar products. We refer to the terms $\{(\theta_j \cdot \eta_i)\}$ as the total *sociodemographic valence* by i for leader j.

In similar fashion, the terms $\{(\alpha_j \cdot \tau_i)\}$ are scalar products, where τ_i is voter i's perception of the trait of leader j, with coefficient α_j. Let $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)$.

The trait score for the election of 2008 was obtained by factor analysis from a set of survey questions, as mentioned above.

---

[40] These will depend on the survey but will include such characteristics as class, domicile, education, income, and religious orientation, etc.

The function $\mu_j(z_j)$ is the sum of other aspects of the valence of leader j. Note that we assume that this aspect of valence is dependent on the leader position, but not on the voter position. In the analysis below, we suggest that this term can be regarded as the effect of activist contributions to the leader. Let $\boldsymbol{\mu} = \{\mu_j\}$.

The term β is a positive constant, called the *spatial parameter*, giving the importance of policy difference defined in terms of a metric induced from the Euclidean distance, $||x_i - z_j||$, between the voter's ideal point, x_i, and the leader position z_j. The vector $\boldsymbol{\varepsilon} = (\varepsilon_1, \ldots, \varepsilon_j, \ldots, \varepsilon_p)$ is the stochastic error, with the multivariate Gumbel (Type I extreme value) distribution. The variance of ε_j is fixed at $\frac{\pi^2}{6}$, so that by definition β has dimension L^{-2}, where L is whatever unit of measurement is used in X.

Various submodels are *pure sociodemographic* (SD), denoted $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta})$, *pure spatial*, $\mathbb{M}(\boldsymbol{\Lambda}, \beta)$, *joint spatial*, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \beta)$, and *joint spatial with traits*, $\mathbb{M}(\boldsymbol{\Lambda}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \beta)$.

As shown in Schofield (2006), the *first-order condition for a local Nash equilibrium under vote maximization* is given by the *balance equation* for each z_j^*:

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j^*) + \frac{1}{2\beta}\frac{d\mu_j}{dz_j}(z_j^*) = 0. \tag{5}$$

Here, the term

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j) \equiv \left[z_j^{\text{el}} - z_j \right].$$

is the *marginal electoral pull of leader j* at the point z_j and can be regarded as a gradient vector, at z_j, pointing toward the weighted electoral mean of the leader. The *weighted electoral mean* for leader j is given by

$$z_j^{\text{el}} \equiv \sum_{i=1}^{n} \varpi_{ij} x_i$$

If $\rho_{ij}(\mathbf{z}) = \rho_{ij}$ is the probability voter i chooses leader j, at $\mathbf{z}$ then the weights are given by the p by n matrix array of weights

$$[\varpi_{ij}] \equiv \left[\frac{[\rho_{ij} - \rho_{ij}^2]}{\sum_{k \in N}[\rho_{kj} - \rho_{kj}^2]} \right] \tag{6}$$

When z_j is equal to the weighted electoral mean then the electoral pull is zero. The gradient vector $\frac{d\mu_j}{dz_j}(z_j)$ is called *the marginal activist pull for leader j* at z_j.

If $\mathbf{z}^* = (z_1^*, \ldots z_j^*, \ldots z_p^*)$ is such that each z_j^* satisfies the balance equation then call $\mathbf{z}^*$ a *balance solution*. We may rewrite (5) as

$$\left[\mathbf{z}^* - \mathbf{z}^{\text{el}} \right] = \frac{1}{2\beta}\frac{d\mu}{d\mathbf{z}}(\mathbf{z}) \tag{7}$$

For the pure spatial model, $\mathbb{M}(\mathbf{\Lambda}, \beta)$, it follows from (6) that, when the leader positions are identical, then $\rho_{kj} = \rho_j$, is independent of the voter k. Thus $\varpi_{ij} = \frac{1}{n}$ for all i and j, gives the first-order condition for a LNE for $\mathbb{M}(\mathbf{\Lambda}, \beta)$. By a change of coordinates, we can choose $z_j^* = \frac{1}{n} \sum_{i=1}^{n} x_i \equiv 0$. It follows that $\mathbf{z}_0 = (0, \ldots 0)$ is a candidate for a LNE for $\mathbb{M}(\mathbf{\Lambda}, \beta)$. Schofield (2007a) shows that the Hessian of the vote share of leader j at $\mathbf{z}_0$ can be identified with the characteristic matrix

$$C_j \equiv 2\beta(1 - 2\rho_j)\nabla_0 - I. \tag{8}$$

Here, I is the identity matrix and ∇_0 is the electoral covariance matrix and $\rho_j \equiv \rho_j(\mathbf{z}_0)$.

The *convergence coefficient* of the model, $\mathbb{M}(\mathbf{\Lambda}, \beta)$, is defined to be

$$c \equiv c((\mathbf{\Lambda}, \beta) \equiv 2\beta[1 - 2\rho_1]\sigma^2 \tag{9}$$

where σ^2 is the total variance, the trace of ∇_0, and $\rho_1 \equiv \rho_1(\mathbf{z}_0)$ for the lowest valence leader.

Note, however, that this argument does not follow for the model $\mathbb{M}(\mathbf{\Lambda}, \boldsymbol{\theta}, \boldsymbol{\alpha}, \beta)$. Even when the leader positions are identical, the probabilities $\{\rho_{kj}\}$ will depend on k. It is necessary, therefore, to compute the vector $\mathbf{z}^{\text{el}} = (z_1^{\text{el}}, z_2^{\text{el}}, \ldots, z_p^{\text{el}})$ as a step to determine the LNE.[41]

The balance solution requires that the electoral and activist gradients are directly opposed, for every leader. If the various activist groups for leader j are given by a family $\{U_{jt} : t \in A_j\}$ of utility functions then we can represent their joint effect by some contract curve. This *contract curve,* generated by the family $\{U_{jt}\}$ of activist utilities, is the locus of points satisfying the gradient equation

$$\sum_{t \in A_j} a_{jt} \frac{dU_{jt}}{dz_j} = 0, \quad \text{where} \quad \sum_{t \in A_j} a_{jt} = 1 \text{ and all } a_{jt} > 0. \tag{10}$$

This in turn implies that the optimal position of leader j will lie on the *balance locus*

$$\left[z_j^{\text{el}} - z_j^*\right] + \frac{1}{2\beta} \left[\sum_{t \in A_j} a_t \frac{dU_t}{dz_j}\right] = 0. \tag{11}$$

The simplest case, discussed in Miller and Schofield (2003), is in two dimensions, where each leader has two activist groups. In this case, the contract curve for each leader will, generically, be a one-dimensional arc. Miller and Schofield (2003) also supposed that the activist utility functions were ellipsoidal, mirroring differing saliences on the two axes. In this case the contract curve for each leader would be a *catenary*, and the balance locus would be a one-dimensional arc. The balance solution for each leader naturally depends on the position(s) of opposed leader(s), and on the

---

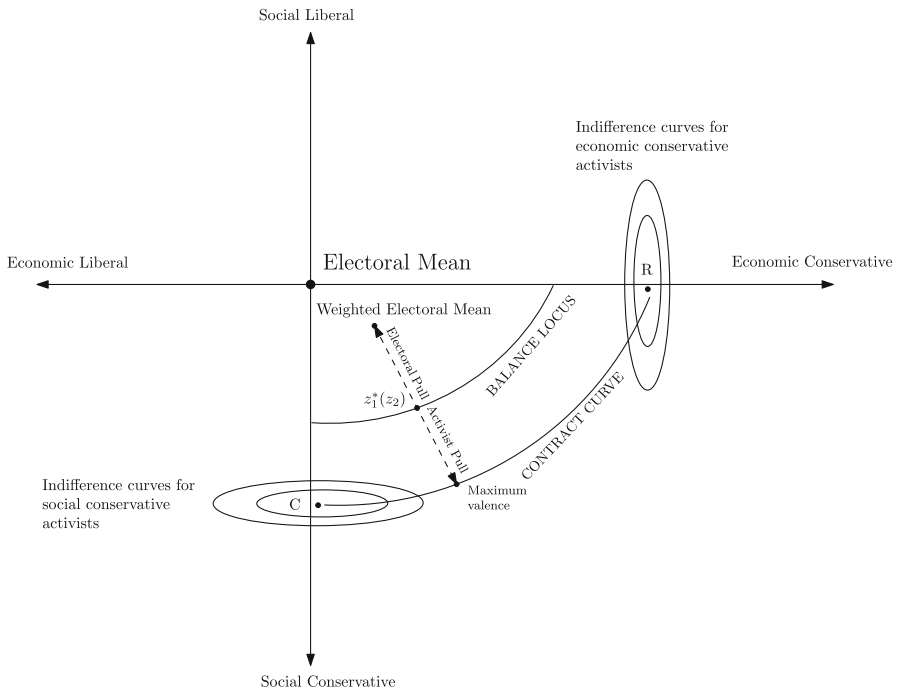[41] We did this using a MATLAB algorithm, based on the gradient of the vote share function.

Fig. 7 Republican contract curve and balance locus

coefficients, as indicated above, of the various activists. These coefficients depend on the willingness of each activist group to supply resources in order to influence the political leader. The determination of the balance solution can be obtained by computing the vote share Hessian along the balance locus. Figure 7 illustrates the balance locus and contract curve for a Republican candidate.

Note that the combination

$$\sum_{t \in A_j} a_k \frac{dU_t}{dz_j}$$

may be interpreted as the marginal utility of the leader of party j, induced by the activist support.

To see this, suppose that each leader were to maximize the function

$$V_j(\mathbf{z}) = \delta \mu_j(z_j) + \frac{1}{n} \Sigma_i \rho_{ij}(\mathbf{z})$$

where μ_j is no longer an activist function, but a policy determined component of the leader's utility function, while δ is the weight given to the policy preference, Then the first-order condition is almost precisely as obtained above, namely

$$\frac{d\mathcal{E}_j^*}{dz_j}(z_j^*) + \frac{\delta'}{2\beta}\frac{d\mu_j}{dz_j}(z_j^*) = 0.$$

Here, $\frac{d\mu_j}{dz_j}(z_j^*)$ is a gradient pointing toward the policy preferred position of the leader and δ' is a product of n and δ, divided by a summation across the voter probabilities. Thus, we can make the identity

$$\delta'\frac{d\mu_j}{dz_j}(z_j^*) = \sum_{t \in A_j} a_t \frac{dU_t}{dz_j}.$$

This equation implies that the leader's marginal policy preference can be identifed with a combination of the marginal preferences of the party activists. To solve this equation in detail requires solving the game between activists and leaders, as outlined in Grossman and Helpman (1994, 1996, 2001). For our purposes, it is sufficient to use this reduced form, as we are interested in the difference $[z_j^{el} - z_j^*]$ between the LNE position, z_j^{el}, of party j and its estimated position, z_j^*.

Appendix 2: question wording for the 2008 American national election study

1. Do you think the government should provide more services than it does now, fewer services than it does now, or about the same number of services as it does now?
2. Do you favor, oppose, or neither favor nor oppose the U.S. government paying for all necessary medical care for all Americans?
3. A proposal has been made that would allow people to put a portion of their Social Security payroll taxes into personal retirement accounts that would be invested in stocks and bonds. Do you favor this idea, oppose it, or neither favor nor oppose it?
   I am going to ask you three questions, and ask you to choose which of two statements in these questions comes closer to your own opinion.
4. One, the main reason government has become bigger over the years is because it has gotten involved in things that people should do for themselves. Two, government has become bigger because the problems we face have become bigger.
5. One we need a strong government to handle today's complex economic problems. Two, the free market can handle these problems without government being involved.
6. One, the less government, the better. Two there are more things that government should be doing.
7. This country would be better if we worried less about how equal people are. Do you agree strongly, agree somewhat, neither agree nor disagree, disagree somewhat, or disagree strongly with this statement?
8. Do you think that big companies should pay a larger percent of their profits in taxes than small businesses do, that big companies should pay a smaller percent

of their profits in taxes than small businesses do, or that big companies and small businesses should pay the same percent of their profits in taxes?

9. Should federal spending on welfare programs be increased, decreased, or kept about the same?
10. Do you favor, oppose, or neither favor nor oppose the U.S. government making it possible for illegal immigrants to become U.S. citizens?
11. Do you think the number of immigrants from foreign countries who are permitted to come to the United States to live should be increased a lot, increased a little, left the same as it is now, decreased a little, or decreased a lot?
12. I'd like to describe a series of circumstances in which a woman might want to have an abortion. For each one, please tell me whether you favor, oppose, or neither favor nor oppose it being legal for the woman to have an abortion in that circumstance.

    1. Staying pregnant would hurt the woman's health but is very unlikely to cause her to die.
    2. Staying pregnant could cause the woman to die.
    3. The pregnancy was caused by sex the woman chose to have with a blood relative.
    4. The pregnancy was caused by the woman being raped.
    5. The fetus will be born with a serious birth defect.
    6. Having the child would be extremely difficult for the woman financially.
    7. The child will not be the sex the woman wants it to be.

13. Do you favor or oppose laws to protect homosexuals against job discrimination?
14. Do you think homosexuals should be allowed to serve in the United States Armed Forces or don't you think so?
15. Do you think gay or lesbian couples, in other words, homosexual couples, should be legally permitted to adopt children?
16. Should same-sex couples be allowed to marry, or do you think they should not be allowed to marry?
17. This country would have many fewer problems if there were more emphasis on traditional family ties. Do you agree strongly, agree somewhat, neither agree nor disagree, disagree somewhat, or disagree strongly with this statement?
18. Do you think the federal government should make it more difficult for people to buy a gun than it is now, make it easier for people, or keep the rules the same?
19. Some people feel that the government in Washington should make every effort to improve the social and economic position of blacks. Others feel that the government should not make any special effort to help blacks because they should help themselves. Where would you place yourself on this scale, or haven't you thought much about this?
20. Irish, Italians, Jewish and many other minorities overcame prejudice and worked their way up. Blacks should do the same without any special favors. Do you agree strongly, agree somewhat, neither agree nor disagree, disagree somewhat, or disagree strongly with this statement?
21. Generations of slavery and discrimination have created conditions that make it difficult for blacks to work their way out of the lower class. Do you agree

strongly, agree somewhat, neither agree nor disagree, disagree somewhat, or disagree strongly with this statement?

22. It's really a matter of some people not trying hard enough; if blacks would only try harder they could be just as well off as whites. Do you agree strongly, agree somewhat, neither agree nor disagree, disagree somewhat, or disagree strongly with this statement?

23. We hear a lot of talk these days about liberals and conservatives. Where would you place yourself on a scale from liberal to conservative?

References

Acemoglu D, Robinson J (2006) Economic origins of dictatorship and democracy. Cambridge University Press, Cambridge

American National Election Study (2008)

Arian A, Shamir M (1999) The election in Israel: 1996. SUNY Press, Albany

Baron D (1994) Electoral competition with informed and uniformed voters. Am Political Sci Rev 88:33–47

Bawn K, Rosenbluth F (2005) Short versus long coalitions: electoral accountability and the size of the public sector. Am J Political Sci 50:251–265

Bayh E (2010) Why I am leaving the Senate. New York Times. February 21.

Benoit K, Laver M (2006) Party policy in modern democracies. Routledge, London

Bernhardt D, Duggan J, Squintani F (2009) The case for responsible parties. Am Political Sci Rev 103:570–587

Clarke HD, Sanders D, Stewart M, Whiteley P (2005) Political choice in Britain. Oxford University Press, Oxford

Clarke HD, Kornberg A, Scotto T (2009a) Making political choices. Toronto University Press, Toronto

Clarke H, Sanders D, Stewart M, Whiteley P (2009b) Performance politics and the British voter. Cambridge University Press, Cambridge

Downs A (1957) An economic theory of democracy. Harper and Row, New York

Duggan J, Fey M (2005) Electoral competition with policy-motivated candidates. Games Econ Behav 51:490–522

Duverger M (1954) Political parties: their organization and activity in the modern state. Wiley, New York

Dworkin R (2010) The "devastating" decision. New York Rev Books 57(3):39

Enelow JM, Hinich M (1982) Nonspatial candidate characteristics and electoral competition. J Politics 44:115–131

Enelow JM, Hinich M (1989) The location of American Presidential candidates. Math Comput Model 12:417–435

Erikson RS, Romero DW (1990) Candidate equilibrium and the behavioral model of the vote. Am Political Sci Rev 84:1103–1126

Fiorina M, Abrams SJ, Pope JC (2005) Culture war?: the myth of a polarized America. Pearson Longman, New York

Grossman GM, Helpman E (1994) Protection for sale. Am Econ Rev 84:833–850

Grossman GM, Helpman E (1996) Electoral competition and special interest politics. Rev Econ Stud 63:265–286

Grossman GM, Helpman E (2001) Special interest groups. MIT Press, Cambridge, MA

Herrera H, Levine D, Martinelli C (2008) Policy platforms, campaign spending and voter participation. J Publ Econ 92:501–513

Jeong G-H, Miller G, Schofield C, Sened I (2010) Cracks in the opposition: Partisan realignment in US Senate immigration negotiations. Working paper, Washington University

Jessee SA (2010) Partisan bias, political information and spatial voting in the 2008 Presidential Election. J Politics 72:327–340

Keefer P (2004) What does political economy tell us about economic development-and vice versa. Annu Rev Political Sci 7:247–272

Laakso M, Taagepera R (1979) Effective number of parties: a measure with applications to West Europe. Comp Political Sci 12:3–27

Laver M, Schofield N (1990) Multiparty Government: the politics of coalition in Europe. Oxford University Press, Oxford. Reprinted 1998, University of Michigan Press, Ann Arbor

McCarty N, Poole K, Rosenthal H (2006) Polarized America. MIT Press, Cambridge, MA

McKelvey RD, Patty JW (2006) A theory of voting in large elections. Games Econ Behav 57:155–180

McKelvey RD, Schofield N (1987) Generalized symmetry conditions at a core point. Econometrica 55: 923–933

Miller G, Schofield N (2003) Activists and partisan realignment in the U.S. Am Political Sci Rev 97: 245–260

Miller G, Schofield N (2008) The transformation of the Republican and Democratic coalitions in the U.S. Perspect Politics 6:433–450

Olmert E (2008) The time has come to say these things. New York Rev Books 55(19):6–8

Olson M (1965) The logic of collective action. Yale University Press, New Haven

Persson T, Tabellini G (2000) Political economics. MIT Press, Cambridge, MA

Persson T, Tabellini G (2003) The economic effect of constitutions. MIT Press, Cambridge, MA

Poole K, Rosenthal H (1984) U.S. Presidential elections 1968–1980: a spatial analysis. Am J Political Sci 28:283–312

Riker WH (1953) Democracy in the United States. Macmillan, New York

Riker WH, Ordeshook PC (1973) An introduction to positive political theory. Prentice-Hall, Englewood Cliffs, NJ

Schofield N (1999) The heart and the uncovered set. J Econ Suppl 8:79–113

Schofield N (2002) Representative democracy as social choice. In: Arrow K, Sen A, Suzumura K (eds) The handbook of social choice and welfare. North Holland, New York

Schofield N (2006) Equilibria in the spatial stochastic model with party activists. Rev Econ Des 10:183–203

Schofield N (2007a) The mean voter theorem: necessary and sufficient conditions for convergent equilibrium. Rev Econ Stud 74:965–980

Schofield N (2007b) Political equilibrium with electoral uncertainty. Soc Choice Welf 28:461–490

Schofield N, Milller G (2007) Elections and activist coalitions in the U.S. Am J Political Sci 51:518–531

Schofield N, Sened I (2006) Multiparty democracy: elections and legislative politics. Cambridge University Press, Cambridge

Schofield N, Zakharov A (2010) A stochastic model of the 2007 Russian Duma election. Pub Choice 142:177–194

Schofield N, Miller G, Martin A (2003) Critical elections and political realignments in the U.S.: 1860–2000. Political Stud 51:217–240

Schofield N, Claassen C, Ozdemir U (2010a) Empirical and formal models of the United States presidential election in 2000. Working paper, Washington University

Schofield N, Claassen C, Ozdemir U, Tavits M (2010b) Modeling elections in the United States and Poland: voter perceptions, political leaders and activists. Working paper, Washington University

Schofield N, Gallego M, Ozdemir U, Jeon J (2010c) Modeling elections in Canada and Great Britain. Working Paper: Washington University in Saint Louis

Schofield N, Gallego M, Ozdemir U, Zakharov A (2011) Competition for popular support: a valence model of elections in Turkey. Soc Choice Welf (this issue). doi:10.1007/s00355-010-0505-2

Serra G (2010) Polarization of what? A model of elections with endogenous valence. J Politics 72:426–427

Stokes D (1963) Spatial models and party competition. Am Political Sci Rev 57:368–377

Stokes D (1992) Valence politics. In: Kavanagh D (ed) Electoral politics. Clarendon Press, Oxford

Sundquist JL (1973) Dynamics of the party system: alignment and realignment of political parties in the United States. The Brookings Institution, Washington, DC

Taylor M, Herman VM (1971) Party systems and government stability. Am Political Sci Rev 65:28–37

Tomasky M (2010) The money fighting health care reform. New York Rev Books 57(6):10–14

Warwick P (1994) Government survival in parliamentary democracies. Cambridge University Press, Cambridge